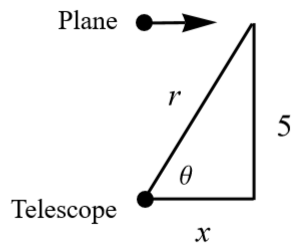
Exercise 45

A plane flies horizontally at an altitude of 5 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\pi/3$, this angle is decreasing at a rate of $\pi/6$ rad/min. How fast is the plane traveling at that time?

Solution

Draw a schematic of the plane's path at a certain time.



$$\left.\frac{d\theta}{dt}\right|_{\theta=\pi/3} = -\frac{\pi}{6}\,\frac{\text{rad}}{\text{min}}$$

The aim is to find dx/dt when $\theta = \pi/3$. Use a trigonometric function to relate the angle θ with convenient sides of the triangle.

$$\tan\theta = \frac{5}{x}$$

Solve for x.

$$x = 5\cot\theta$$

Take the derivative of both sides with respect to time by using the chain rule.

$$\frac{d}{dt}(x) = \frac{d}{dt}(5\cot\theta)$$

$$\frac{dx}{dt} = (-5\csc^2\theta)\cdot\frac{d\theta}{dt}$$

Therefore, at the time when the angle of elevation is $\pi/3$, the plane is travelling at

$$\left.\frac{dx}{dt}\right|_{\theta=\pi/3} = \left(-5\csc^2\frac{\pi}{3}\right)\cdot\left(-\frac{\pi}{6}\right) = \frac{10\pi}{9}\,\frac{\text{km}}{\text{min}} \approx 3.49066\,\frac{\text{km}}{\text{min}}.$$